Another angle on ANGLES

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describes a dynamic approach used to develop the concept of angle as a measure of turn.

The impact of written and printed texts on the geometry curriculum has a very long history. Static diagrams, static definitions and static proofs have dominated our classrooms for centuries. Geometry is about shape and space, and about the relationships between shapes in space. Movement is a key aspect of those relationships but movement cannot be shown in a printed text book, so it is relegated to a minor role in the primary mathematics curriculum if we are lucky, or ignored altogether if we are not.

One area where print has had a particularly significant effect is in the teaching and learning of the concept of angle. An angle is a measure of turn. It is a measure of movement, not a shape. However, we cannot show the movement on a printed page. At best, we can indicate it with an arrowed arc (Figure 1). Even this is often reduced to a simple arc (Figure 2) or it is left out altogether (Figure 3).

Students whose experience of angles is limited to those shown in a text book may develop a misconception about angle size. They come to believe that the size of an angle relates to the length of the lines that represent the turn, not to the turn itself. They may think that an angle represented by two longer lines is greater than one represented by two shorter lines regardless of the angle between them (Clausen-May 2005, p. 69; see Figure 4).

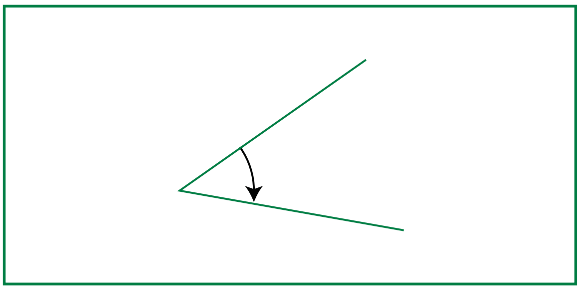

Figure 1

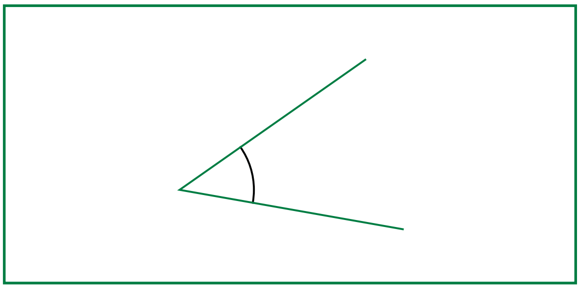

Figure 2

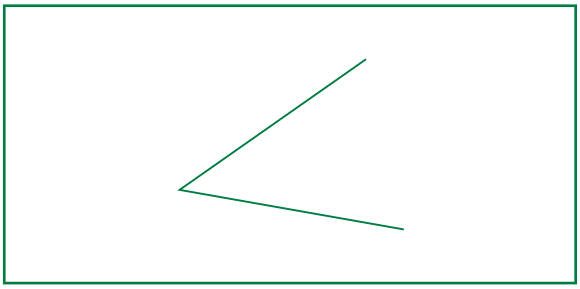

Figure 3

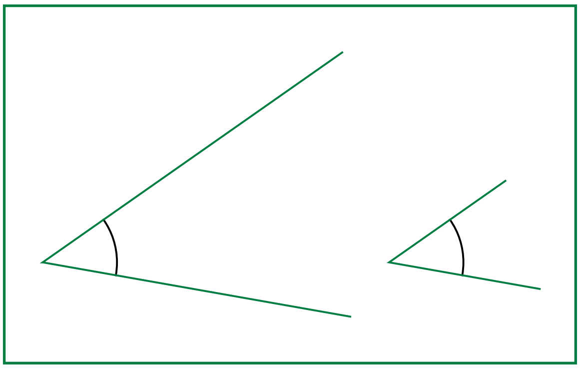

Figure 4

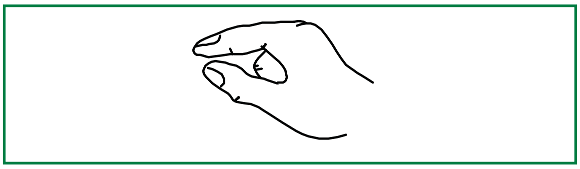

Figure 5

You might try an experiment with your students. Ask them to show you one degree. You may find that many of them show you a linear distance (Figure 5). It may be a very small distance — but it will identify a basic misconception that has been introduced and reinforced by our dependence on printed diagrams. These students may not realise that a degree is just one, very small, fraction of a turn. They may understand the concept of a quarter turn and a half turn — perhaps they have practised turning through such fractions of a turn themselves, in the gym hall or the play ground even if not in the mathematics classroom. Nevertheless, they may not relate the dynamic action of turning to the static printed diagrams in a mathematics text book.

The dynamic graphics that can be provided quite easily with a computer offer an alternative way to develop children's understanding of angle. This emphasises the crucial link between fractions of a turn and the measure of an angle. It lifts angles out of static diagrams, and establishes them as a dynamic concept based securely on rotation. A set of three PowerPoint presentations which explain and develop this concept may be downloaded from
www.aamt.edu.au/
Professional-learning/Professional-reading
Please note that the presentations require the 2003 (or later) version of PowerPoint to operate correctly.

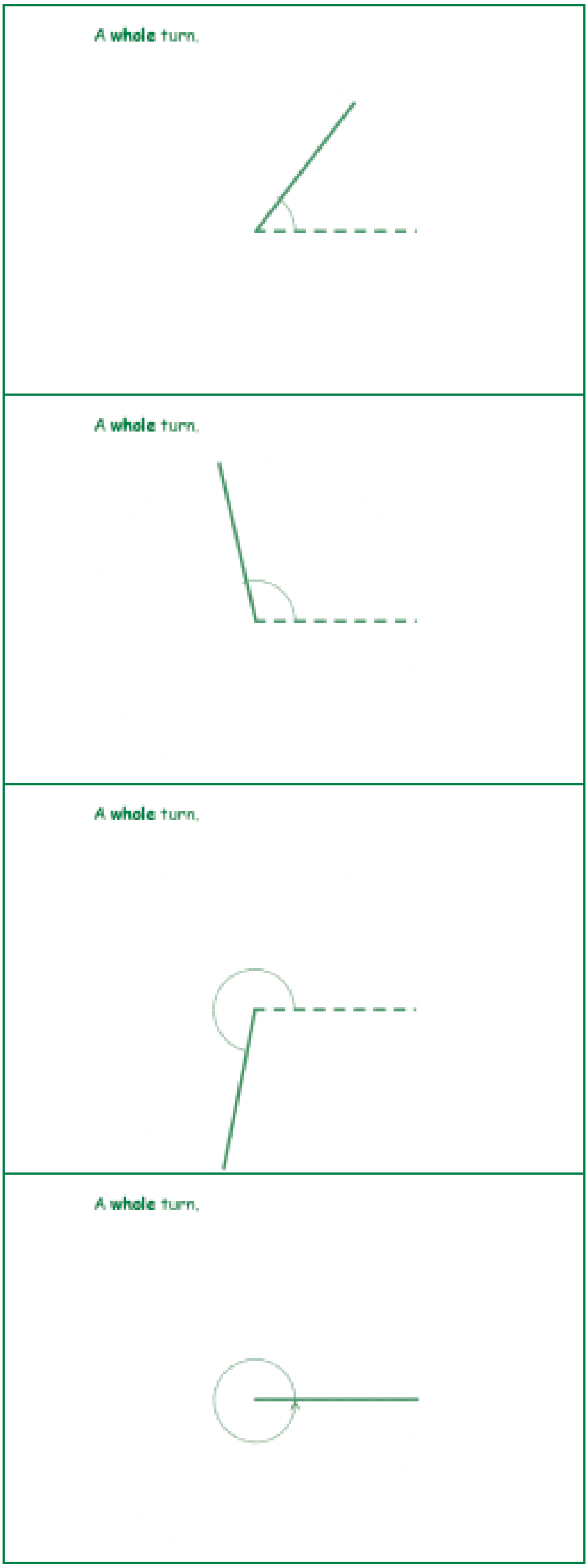

Figure 6

The image of a whole turn is presented first (Figure 6), with a line segment making a full rotation leaving an arrowed arc in its wake.

This is followed by different fractions of a turn, and by the definitions of right, acute, obtuse, straight and reflex angles (Figures 7–8).

Only in the second presentation is the idea of a degree introduced — as just another, very small, fraction of a turn (Figure 9).

Then the link is made between the 360° of a whole turn, and the measure in degrees of other fractions of a turn: a quarter-turn, a half-turn, a three-quarter-turn, a sixth-turn, and so on (Figure 10).

In the third presentation this view of angle as a measure of turn is extended to an understanding of the internal angles of a polygon. Turning through the three internal angles of a triangle, one after another, leaves the "pointer" facing in the opposite direction to its starting position (Figure 11).

So the internal angles of a triangle add up to a half turn. This dynamic image — of a pointer turning through the angles of a triangle — is one that pupils can recall, and use to check the angle sum of any triangle. In the same way, the angle sum of other polygons may be checked by turning a pencil, or similar object, through each of the angles in succession.

So a dynamic approach to the concept of angle as a measure of turn offers an insight into the properties of polygons that goes well beyond the "rules" for finding their angle sums. Turning a pointer through the internal angles of a polygon, and watching how many half turns it completes, will help students to understand that the sum of its angles is equal to one half turn (or 180°), multiplied by the number of sides minus two. More importantly perhaps, this approach will help them to appreciate *why* the formula works. It will offer a "picture in the mind" which they can use to help them to recall the angle sum of any polygon, and to see how it is derived — even if the formula itself has, as sometimes happens, slipped their memory.

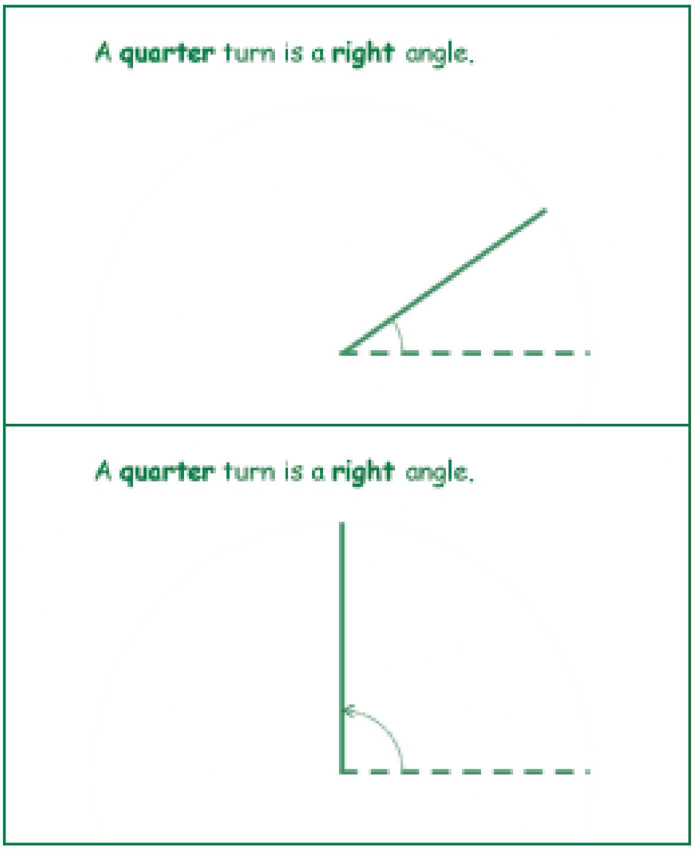

Figure 7

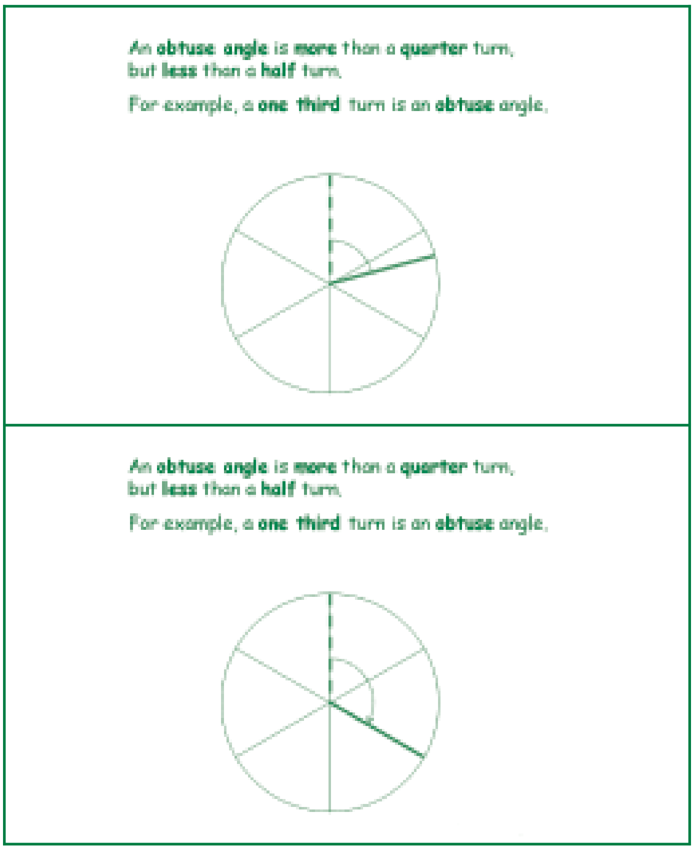

Figure 8

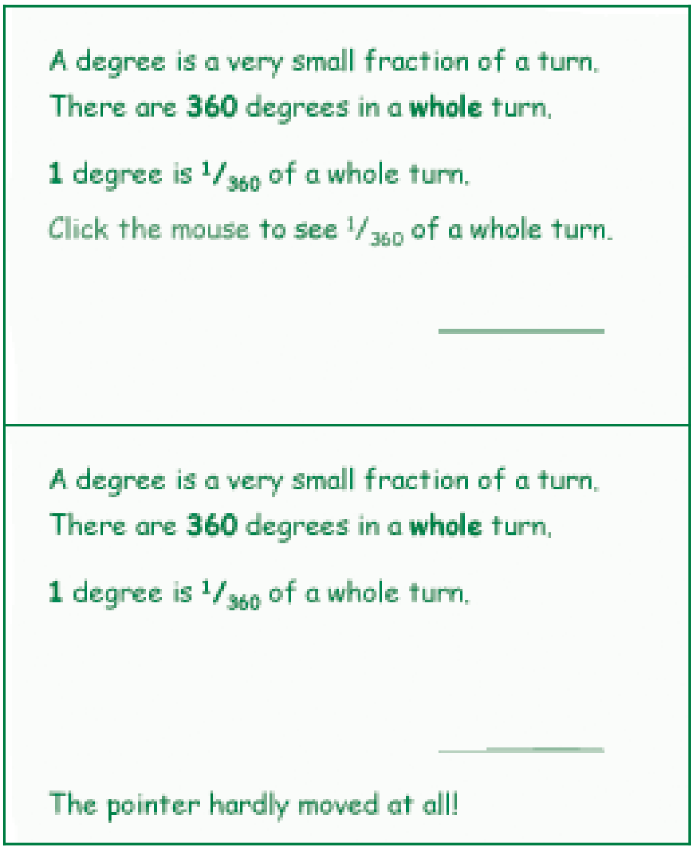

Figure 9

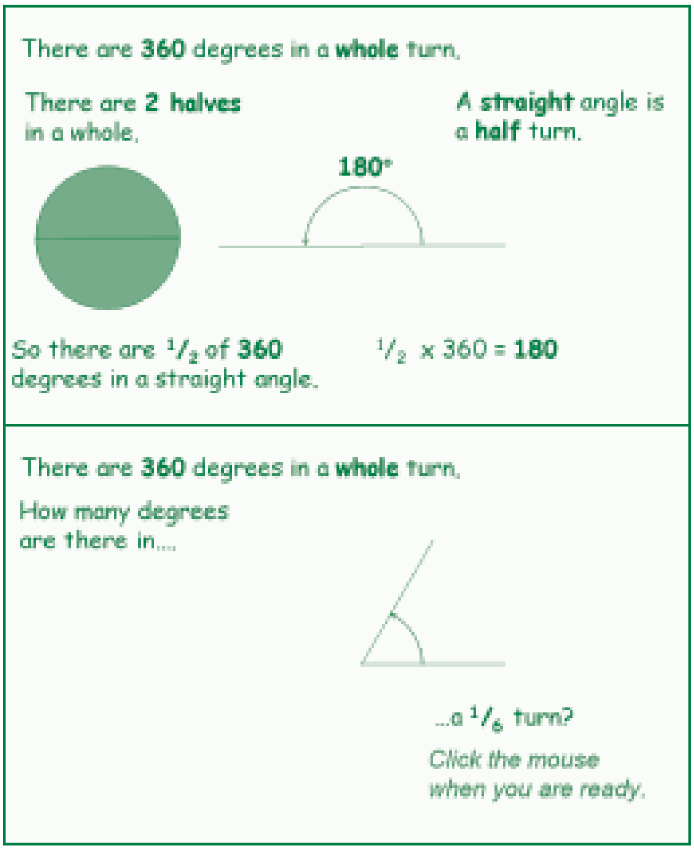

Figure 10

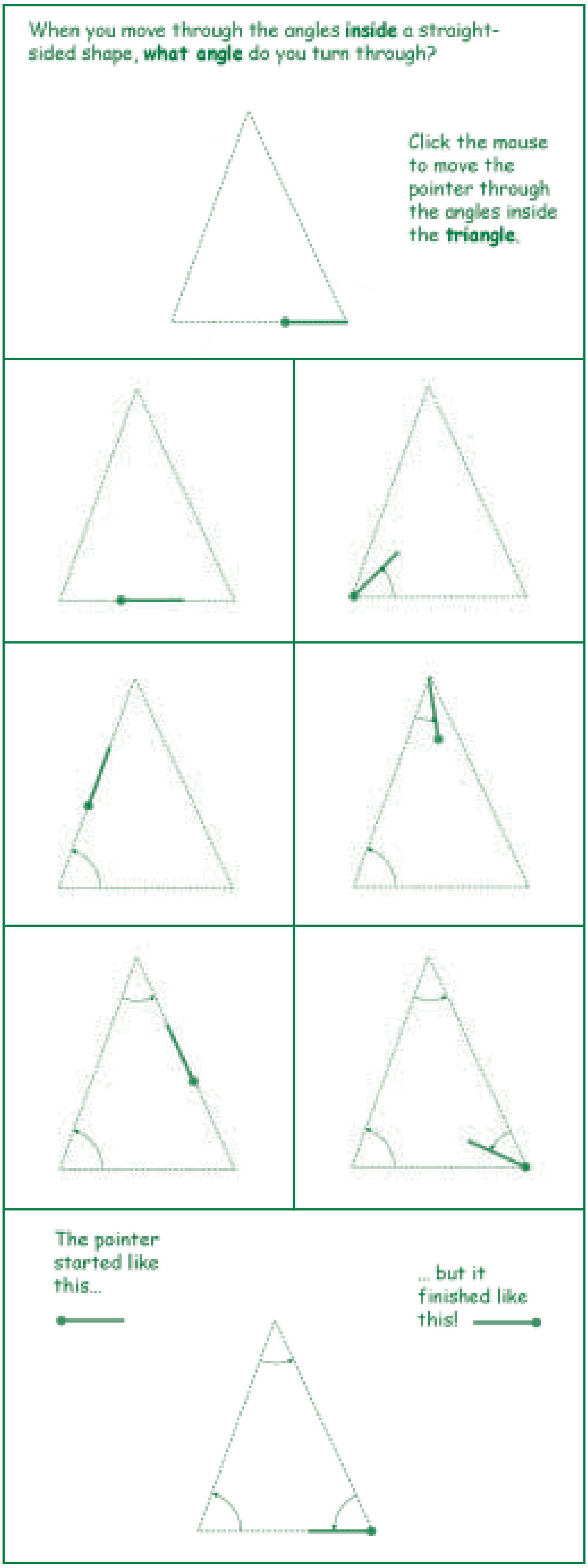

Figure 11

These ideas are inevitably difficult to convey in print, for, as Mackrell and Johnston-Wilder (2005, p. 82) remark,

> One of the ironies of trying to describe motion and its effects in text is that one necessarily has to miss out on *all* of the essential ingredients. Not least among these is the sense of surprise and wonder that animating mathematical diagrams and images can bring, externalising and setting back in motion images that have been held static in the pages of textbooks for over 2000 years.

For at least some of our pupils, the dynamic images may offer a meaningful — and therefore memorable — route to the key concept of angle. Computers are beginning to break the stranglehold of the text-based curriculum that has dominated mathematics for so many years. At long, long last, movement is back.

References

Clausen-May, T. (2005). *Teaching Maths to Pupils with Different Learning Styles.* London: Paul Chapman.

Mackrell, K. & Johnston-Wilder, P. (2005). Thinking geometrically. In S. Johnston-Wilder & D. Pimm (Eds), *Teaching Secondary Mathematics with ICT.* Open University Press.

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